PMM U.S.S.R.,Vol.45,pp.459-463 0021-8928/82/4 0459 $7.50/0

UDC 532/534

THE METHOD OF MOVING COORDINATES IN PROBLEMS OF CONTINUOUS MEDIUM MECHANICS*

A.E. MEDVEDEV and V.M. FOMIN

The method of difference grid construction is used in conjunction with the solution of a physical problem /1/ for solving elastoplastic and gasdynamic problems. Two different variational criteria of coordinate grid selection are considered. The first is based on the investigation of a functional dependent on the flow velocity and motion of moving curvilinear coordinates. An estimate is obtained of the dependence of curvilinear coordinate behavior on the local flow properties. The second criterion is based on the dependence of the functional on the difference scheme variance, and is obtained using the properties of solution of the Hopf equation, as a model. This makes it possible to take into account not only the properties of solution but, also, those of the difference scheme.

In solving multidimensional problems of mechanics of continuous media with free boundaries and large deformations it is necessary to use numerical algorithms that take into account the a priori known basic singularities of the solution. A method of difference grid construction in conjunction with the solution of the problem was proposed in /1/, and was later applied in /2,3/ for solving some very simple problems of gasdynamics.

1. Equations of elastoplasticity in moving curvilinear coordinates and the variational criterion of the construction of a grid dependent on the flow.

Consider the one-to-one mapping $\mathbf{x} = \mathbf{x}(t, \mathbf{q})$ of space $G = \{t, q^1, q^2, q^3\}$ onto space $X = \{t, x^1, x^2, x^3\}$, which retains the separation of space coordinates and time. Mapping $\mathbf{x} = \mathbf{x}(t, \mathbf{q})$ defines the moving curvilinear coordinate grid, and vector $\mathbf{w} = \partial\mathbf{x}/\partial t$ of its velocity in space X.

We use the notation

$$a_j^i = \frac{\partial x^i}{\partial q^j}, \quad b_j^i = \frac{\partial q^i}{\partial x^j}, \quad a_\beta^i b_j^\beta = \delta_j^i, \quad \Delta = \det\| a_j^i \|, \quad \Gamma_{\alpha\beta}^i = \left(\frac{\partial}{\partial q^\beta} a_\alpha^k\right) b_k^i \quad (\alpha, \beta, i, j, k = 1, 2, 3)$$

where summation is carried out over the recurring index. The equations defining the three-dimensional motion of a perfect elastoplastic medium are in Cartesian coordinates in X of the form

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x^k}(\rho u^k) = 0, \quad \frac{\partial\rho u^i}{\partial t} + \frac{\partial}{\partial x^k}(\rho u^i u^k + P\delta_i^k - S^{ik}) = 0$$

$$\frac{\partial}{\partial t}\left[\rho\left(e + \frac{u^2}{2}\right)\right] + \frac{\partial}{\partial x^k}\left[\rho u^k\left(e + \frac{u^2}{2} + \frac{P}{\rho}\right) - S^{ik}u^i\right] = 0$$

$$\frac{dS^{ij}}{dt} - \frac{\partial}{\partial x^k}\left[\mu(u^i\delta_j^k + u^j\delta_i^k) - \frac{2}{3}u^k\delta_j^i\right] = 0 \quad (i, j, k = 1, 2, 3)$$

where $\mathbf{u}$ is the velocity vector, μ is the shear modulus, pressure $P = P(\rho, e)$ is a known function, and S^{ij} are components of the stress tensor deviator.

We use the Mises form $S^{ij}S^{ij} \leqslant {}^2/_3\sigma_s{}^2$ of plasticity condition.

Representing the laws of conservation for the first C^α and second $E^{\alpha\beta}$ rank tensors /6/, where in this case

$$C^\circ = \rho\left(e + \frac{u^2}{2}\right), \quad C^i = \rho\left(e + \frac{u^2}{2}\right)u^i + Pu^i - S^{ki}u^k$$

$$\| E^{\alpha\beta} \| = \begin{Vmatrix} \rho & \rho u^k \\ \rho u^i & \rho u^i u^k + P\delta_i^k - S^{ik} \end{Vmatrix} \quad (i, k = 1, 2, 3)$$

and the formula for the material derivative in coordinates G, we obtain in moving curvilinear coordinates the following equations of perfect elastoplasticity:

*Prikl.Matem.Mekhan.,45,No.4,624-630,1981

$$\frac{\partial \rho\Delta}{\partial t} + \frac{\partial}{\partial q^k}[\rho\Delta(v^k - \omega^k)] = 0 \tag{1.1}$$

$$\frac{\partial \rho\Delta(v^i - w^i)}{\partial t} + \frac{\partial}{\partial q^k}\{\Delta[\rho(v^i - \omega^i)(v^k - \omega^k) - \xi^{ik}]\} +$$

$$\Delta\rho\left[\frac{\partial \omega^i}{\partial t} + \frac{\partial \omega^i}{\partial q^k}(2v^k - \omega^k) + \left(v^k v^l - \frac{1}{\rho}\xi^{kl}\right)\Gamma^i_{kl} + \frac{1}{\rho}g^{ik}\frac{\partial P}{\partial q^k}\right] = 0$$

$$\frac{\partial}{\partial t}\left[\rho\Delta\left(e + g_{nm}\frac{v^n v^m}{2}\right)\right] + \frac{\partial}{\partial q^k}\left\{\Delta\left[\rho\left(e + g_{nm}\frac{v^n v^m}{2}\right) \times (v^k - \omega^k) + Pv^k - \xi^{rl}a_l^n v^s g_{rs}\right]\right\} = 0$$

$$\frac{1}{\Delta}\frac{\partial}{\partial t}(\Delta\xi^{ml}a_m{}^i a_l{}^j) - \frac{1}{\Delta}\frac{\partial}{\partial q^k}(\Delta\omega^i\xi^{ml}a_m{}^n a_l{}^j) + \Delta v^k\left[\frac{\partial}{\partial q^k}(\xi^{ml}a_m{}^i a_l{}^j) + \Gamma_{kn}{}^i\xi^{ml}a_m{}^n a_l{}^j + \Gamma_{kn}{}^i\xi^{ml}a_m{}^i a^n{}_l\right] -$$

$$\mu\left\{\left[\delta_j{}^k\left(\frac{\partial \omega^m}{\partial q^n}a_m{}^i + \omega^m\frac{\partial a_m{}^i}{\partial q^n}\right) + \delta_i{}^k\left(\frac{\partial \omega^m}{\partial q^n}a_n{}^j + \omega^m\frac{\partial a_m{}^j}{\partial q^n}\right)\right]b_k{}^n - \frac{2}{3}\delta_k{}^i\frac{\partial \Delta v^k}{\partial q^k}\right\} = 0$$

$$g^{ij} = b_k{}^i b_k{}^j,\quad g_{ij} = a_i{}^k a_j{}^k,\quad v^i = u^k b_k{}^i,\quad \omega^i = w^k b_k{}^i$$
$$\xi^{ij} = S^{kl}b_k{}^i b_l{}^j,\quad \Delta = \det\|a_j{}^i\|\ (i, j, k, l, m, n, r, s = 1, 2, 3)$$

This mapping is one-to-one, when the Jacobian of transformation $\Delta = \det\|a_j{}^i\| \neq 0$. In the system of Eqs. (1.1) it is generally necessary to specify the vector function $\mathbf{w} = \partial\mathbf{x}/\partial t$. But, when solving specific boundary value problems by the method of finite differences, it is convenient to consider the determination of $\mathbf{w}$ in the course of solution for each fixed instant of time.

With this in view we apply the idea of /1/ and determine the vector function $\mathbf{w}$ using the variational principle. We compose the functional

$$\Phi(\mathbf{w}) = \int_{\Omega_t} F\,d\Omega_t \tag{1.2}$$

$$F = \left(\mathbf{w} - \frac{\partial\Psi}{\partial\mathbf{x}}\mathbf{u}\right)^2 + [\varepsilon_1(\operatorname{div}\mathbf{w})^2 + \varepsilon_2\Gamma^2 + \varepsilon_3(\operatorname{rot}\mathbf{w})^2] \times \left[\left(\rho T\frac{dS}{dt} + \operatorname{div}\mathbf{g}\right) + c\right] + \left[k_*\operatorname{div}\mathbf{w} - \operatorname{div}\left(\frac{\partial\Psi}{\partial\mathbf{x}}\mathbf{u}\right)\right]^2 \tag{1.3}$$

where Ω_t is the region occupied by the medium at the instant of time t, and $\varepsilon_1, \varepsilon_2, \varepsilon_3, c, k_*$ are nonnegative constant normalizing multipliers, and $\mathbf{g}$ is the heat influx vector.

The curvilinear coordinates $\mathbf{q}$ of form $\mathbf{q} = \Psi(\mathbf{x})$ are selected at the initial instant of time $t = 0$ depending on the region Ω_0 occupied by the medium.

The first term in (1.3) defines the deviation from Lagrangian coordinates. The second term is the product of two factors of which the first is the sum of squares of volume variation $(\operatorname{div}\mathbf{w})^2$ of shear

$$\Gamma^2 = 4\left[\sum_{i=1}^{3}\left(\frac{\partial w^i}{\partial x^i}\right)^2 + \left(\frac{\partial w^1}{\partial x^1}\frac{\partial w^2}{\partial x^2} + \frac{\partial w^1}{\partial x^1}\frac{\partial w^3}{\partial x^3} + \frac{\partial w^2}{\partial x^2}\frac{\partial w^3}{\partial x^3}\right)\right] + 3\left[\left(\frac{\partial w^1}{\partial x^2} + \frac{\partial w^2}{\partial x^1}\right)^2 + \left(\frac{\partial w^1}{\partial x^3} + \frac{\partial w^3}{\partial x^1}\right) + \left(\frac{\partial w^2}{\partial x^3} + \frac{\partial w^3}{\partial x^2}\right)^2\right]$$

and of rotation $(\operatorname{rot}\mathbf{w})^2$ of a coordinate grid cell, and the second defines energy dissipation in the medium. The second term stipulates that the coordinate grid deformation is to be the lesser the higher is the energy dissipation in the medium. The last term in (1.3) defines the coordinate grid density in the region of velocity gradients $\mathbf{u}$.

The problem of determination of the unknown function $\mathbf{w}$ can be formulated thus: determine among the class of admissible functions in region $\Omega_t(t, q^1, q^2, q^3)$ that which yields the minimum of functional (1.2) with the differential condition (1.1) and the boundary condition

$$\mathbf{w}|_{\partial\Omega_t} = (\partial\Psi/\partial\mathbf{x})\,\mathbf{u}|_{\partial\Omega_t}$$

Note that the problem of determining the vector function $\mathbf{w}$ for gasdynamic flows is similarly formulated, except that the differential conditions (1.1) are in this case of the form /1,6/

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial q^k}[\rho\Delta(v^k - \omega^k)] = 0 \tag{1.4}$$

$$\frac{\partial\rho\Delta(v^i - \omega^i)}{\partial t} + \frac{\partial}{\partial q^k}[\rho\Delta(v^i - \omega^i)(v^k - \omega^k)] + \Delta\rho\left[\frac{\partial\omega^i}{\partial t} + \frac{\partial\omega^i}{\partial q^k}(2v^k - \omega^k) + v^k v^l\Gamma_{kl}{}^i + \frac{g^{ik}}{\rho}\frac{\partial P}{\partial q^k}\right] = \Delta\rho F^i$$

$$\frac{\partial\rho\Delta\left(e + g_{km}\frac{v^k v^m}{2}\right)}{\partial t} + \frac{\partial}{\partial q^j}\left[\Delta\rho\left(e + g_{km}\frac{v^k v^m}{2}\right) \times (v^j - \omega^j) + \Delta P v^j\right] = \Delta\rho g_{km}F^k v^m$$

$$e = e(P, \rho) \quad (i, j, k, l, m = 1, 2, 3)$$

where $\mathbf{F}$ is the vector of mass forces.

2. Examples of coordinate grid dependence on flow properties. Example 1. Consider one-dimensional unsteady flows of perfect gas free of shock waves when $\rho T dS/dt + \operatorname{div} \mathbf{g} \equiv 0$, i.e. without energy dissipation. The Euler equation which ensures the minimum of functional (1.2) under conditions (1.4) with $i, j, k, l, m = 1$ in region $x_0(t) \leqslant x \leqslant x_1(t)$, $t_0 \leqslant t \leqslant t_1$ is of the form

$$\alpha^2 w_{xx} - w = k_* \varphi_{xx} - \varphi - \alpha^2 \tag{2.1}$$

$$\varphi = \frac{\partial \Psi}{\partial x} u(x, t), \quad \alpha^2 = (\varepsilon_1 + 4\varepsilon_2) c + k_*$$

with boundary conditions

$$w|_{x_0(t)} = \varphi|_{x_0(t)} = \varphi_0(t), \quad w|_{x_1(t)} = \varphi|_{x_1(t)} = \varphi_1(t)$$

Solving Eq. (2.1) we obtain

$$w(x, t) = \left(1 - \frac{k_*}{\alpha^2}\right)\left(\operatorname{sh}\frac{x_1 - x_0}{\alpha}\right)^{-1}\left[\varphi_0(t)\operatorname{sh}\frac{x_1 - x}{\alpha} + \varphi_1(t)\operatorname{sh}\frac{x - x_0}{\alpha} + \frac{1}{\alpha}\operatorname{sh}\frac{x - x_0}{\alpha}\int_{x_0}^{x_1}\varphi(\xi, t)\operatorname{sh}\frac{x_1 - \xi}{\alpha}\,d\xi\right] +$$

$$\alpha^2\left[1 - \left(\operatorname{sh}\frac{x_1 - x}{\alpha} + \operatorname{sh}\frac{x - x_0}{\alpha}\right)\left(\operatorname{sh}\frac{x_1 - x_0}{\alpha}\right)^{-1}\right] + \frac{k_*}{\alpha^2}\varphi(x, t) - \left(1 - \frac{k_*}{\alpha^2}\right)\frac{1}{\alpha}\int_{x_0}^{x}\varphi(\xi, t)\operatorname{sh}\frac{x - \xi}{\alpha}\,d\xi$$

When $k_* = \alpha^2$, the following theorem is valid.

Theorem. When in the calculation of one-dimensional unsteady flows free of shock waves of perfect gas the normalizing multipliers satisfy the conditions $(\varepsilon_1 + 4\varepsilon_2) < 1/4$, $k_* = 1/2 \pm 1/2 (1 - 4c(\varepsilon_1 + 4\varepsilon_2))^{1/2}$, the Lagrangian coordinates are optimal, i.e. $w(x, t) = \varphi(x, t)$. The coordinate grid $x = x(t, q)$ is determined by solving the equation with initial condition $dx/dt = w(x, t)$, $x|_{t=t_0} = \Psi^{-1}(q)$.

The Jacobian of transformation $\Delta = \Delta(x, t)$ then satisfies the equation with initial condition

$$\frac{\partial \Delta}{\partial t} - w\frac{\partial \Delta}{\partial x} = \Delta\frac{\partial w}{\partial x}, \quad \Delta|_{t=t_0} = \frac{1}{\Psi'(x)} \tag{2.2}$$

Solving Eq. (2.2) we obtain

$$\Delta = \frac{\exp[f_2(t) - f_2(t_0)]}{\Psi'(x - [f_1(t) - f_1(t_0)])} \tag{2.3}$$

$$f_2(t) = \int \frac{\partial w(f_1(t), t)}{\partial f_1}\,dt$$

where $f_1(t)$ is determined by the solution of equation $df_1/dt = w(f_1, t)$.

It follows from (2.3) that in the region of solution $\Delta(x, t) \neq 0$.

Example 2. When solving equations of perfect gasdynamics in the presence of shock waves the scheme of rippling through computation is commonly used with the addition pseudoviscosity supplement to the pressure

$$P_* = P + \gamma\,\partial u/\partial x$$

where γ is a constant coefficient. The energy dissipation in the gas (without heat influx) is then defined by formula $\gamma(\partial u/\partial x)^2$.

Taking this into account, we express Euler's equation for the minimum of functional (1.2) under conditions (1.4) in region $(x_0(t) \leqslant x \leqslant x_1(t),\ 0 \leqslant t < t_*)$ and in coordinates X in the form

$$h w_{xx} + h_x w_x - w = k_* u_{xx} - u - h \tag{2.4}$$

$$h = \gamma\varepsilon u_x^2 + \alpha^2, \ \varepsilon = \varepsilon_1 + 4\varepsilon_2, \ \alpha^2 = c\varepsilon + k_*^2$$

with boundary conditions

$$w|_{x_0(t)} = u|_{x_0(t)} = u_0(t), \quad w|_{x_1(t)} = u|_{x_1(t)} = u_1(t)$$

Let at a fixed instant of time t the specified solution $u(x, t)$ be a function that is twice continuously differentiable with respect to the variable x over the segment $x \in [x_0(t), x_1(t)]$.

To investigate the behavior of function $w(x, t)$ we introduce the new variable

$$\xi(x) = \beta\int_{x_0}^{x}\frac{dx}{h(x)}, \quad \beta = \left(\int_{x_0}^{x_1}\frac{dx}{h(x)}\right)^{-1}$$

with ξ varying from 0 to 1. Let

$$w(x) = -y(\xi) + \int_{x_0}^{x} \frac{k_* u_x}{h(x)}\, dx + u_0$$

where $y(\xi)$ is the new unknown function that satisfies the equation with the boundary conditions

$$y_{\xi\xi} = \frac{h(\xi)}{\beta^2} y + \frac{h(\xi)}{\beta^2 \sqrt{\gamma\varepsilon}} \int_0^{\xi} (h - k_*) \sqrt{h - \alpha^2}\, d\xi + \frac{h^2(\xi)}{c^2} \tag{2.5}$$

$$y(0) = 0, \quad y(1) = \frac{k_*}{c\sqrt{\gamma\varepsilon}} \int_0^1 \sqrt{h - \alpha^2}\, d\xi - u_1 + u_0$$

If we set $k_* \leqslant \alpha^2$, i.e. $k_*^2 - k_* + c\varepsilon \geqslant 0$, then $h - k_* \geqslant 0$ and the integral in the right-hand side of Eq.(2.5) is a nonnegative function. Estimating the right-hand side of Eq.(2.5) from above and below, we obtain respective estimates for function $w(x)$ of form $w_-(\xi) \leqslant w(\xi) \leqslant w_+(\xi)$, where

$$w_+(\xi) = u(\xi) + h_* \left[(\operatorname{ch}\sigma_* - 1)\frac{\operatorname{sh}\sigma_*\xi}{\operatorname{sh}\sigma_*} - (\operatorname{ch}\sigma_*\xi - 1)\right] + \frac{1}{\beta\sqrt{\gamma\varepsilon}}\left(\left\{\xi - \frac{\operatorname{sh}\sigma_*\xi}{\operatorname{sh}\sigma_*}\int_0^1 \Psi_*(\xi)\, d\xi + \left[\int_0^{\xi} \Psi_*(\xi)\, d\xi - \xi\int_0^1 \Psi_*(\xi)\, d\xi\right\}\right]\right.$$

$$w_-(\xi) = u(\xi) + h_0 \left[(\operatorname{ch}\sigma_0 - 1)\frac{\operatorname{sh}\sigma_0\xi}{\operatorname{sh}\sigma_0} - (\operatorname{ch}\sigma_0\xi - 1)\right] + \frac{1}{\beta\sqrt{\gamma\varepsilon}}\left\{\left(\xi - \frac{\operatorname{sh}\sigma_0\xi}{\operatorname{sh}\sigma_0}\right)\int_0^1 \Psi_0(\xi)\, d\xi + \left[\int_0^{\xi} \Psi_0(\xi)\, d\xi - \xi\int_0^1 \Psi_0(\xi)\, d\xi\right]\right\}$$

$$h_* = \max h(\xi), \quad h_0 = \min h(\xi), \quad \sigma_* = h_*^{1/2}/\beta, \quad \sigma_0 = h_0^{1/2}/\beta$$
$$\Psi_*(\xi) = (h_* - k_*)(h_* - \alpha^2)^{1/2} - (h - k_*)(h - \alpha^2)^{1/2}$$
$$\Psi_0(\xi) = (h_0 - k_*)(h_0 - \alpha^2)^{1/2} - (h - k_*)(h - \alpha^2)^{1/2}$$

The above reasoning enables us to formulate the following theorem.

Theorem. In the calculation of one-dimensional flows of perfect gas with pseudoviscosity added to pressure $P_* = P + \gamma\, \partial u/\partial x$ the estimate

$$w_-(\xi(x)) \leqslant w(x) \leqslant w_+(\xi(x))$$

of the rate of contraction of moving curvilinear coordinates is valid for the known flow $u(x)$ with the constraints introduced above, provided the normalizing multipliers in functional (1.2) satisfy the inequality $k_*^2 - k_* + c\varepsilon \geqslant 0$.

Remark. For one-dimensional equations that define the behavior of an elastoplastic medium Euler's equation (2.4) of the minimum of functional (1.2) retains its form, except that

$$h = -\sigma_x \frac{\partial u}{\partial x}\varepsilon + c\varepsilon + k_*^2$$

where σ_x is the stress tensor component.

By a suitable selection of constants c, ε, k it is possible to have $h(x) \geqslant 0$ for $x_0(t) \leqslant x \leqslant x_1(t)$, hence this theorem is also valid for elastoplastic flows.

Example 3. Consider the construction of functional (1.2) based on the minimization of the error of solution of the basic boundary value problem. To simplify exposition the investigation is carried out on the example of solution of the equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0 \tag{2.6}$$

which has the self-similar solution $u = x/t$ in the domain

$$D\,(x_1(t) \leqslant x \leqslant x_2(t), \quad 0 < t_0 \leqslant t < \infty)$$

In curvilinear coordinates Eq.(2.6) is of the form

$$\frac{\partial x}{\partial q}\frac{\partial u}{\partial t} + \left(u - \frac{\partial x}{\partial t}\right)\frac{\partial u}{\partial q} = 0 \tag{2.7}$$

where $x = x(t, q)$ is the unknown moving curvilinear grid. To solve Eq.(2.7) we use the Lax type difference scheme whose first differential approximation can be represented in the form

$$\frac{\partial x}{\partial q}\frac{\partial u}{\partial t} + \left(u - \frac{\partial x}{\partial t}\right)\frac{\partial u}{\partial q} = \left(\mu\frac{\partial^2 u}{\partial q^2} - \frac{\tau}{2}\frac{\partial^2 u}{\partial t^2}\right)\frac{\partial x}{\partial q} - \left(\mu\frac{\partial^2 x}{\partial q^2} - \frac{\tau}{2}\frac{\partial^2 x}{\partial t^2}\right)\frac{\partial u}{\partial q}, \quad \mu = \frac{h^2}{2\tau}$$

which enables us to represent functional (1.2) as

$$\Phi = \int_\Omega F^2 d\Omega \tag{2.8}$$

$$F = \left(\mu \frac{\partial^2 u}{\partial q^2} - \frac{\tau}{2}\frac{\partial^2 u}{\partial t^2}\right)\frac{\partial x}{\partial q} - \left(\mu \frac{\partial^2 x}{\partial q^2} - \frac{\tau}{2}\frac{\partial^2 x}{\partial t^2}\right)\frac{\partial u}{\partial q}$$

where h and τ are the space and time difference pitches, respectively.

Hence for the determination of $x = x(t, q)$ we can formulate the following problem: among the class of admissible functions $x(t; q)$ determine in the domain $\Omega\,(q(t) \leqslant q \leqslant q_2(t), 0 < t_0 \leqslant t < \infty)$ that which provides the minimum of functional (2.8) with the differential constraint (2.7) and the boundary and initial conditions

$$t = t_0,\quad x = q,\quad q = q_i(t),\quad \partial x/\partial t = u \quad (i = 1, 2)$$

where $q_i(t)$ is obtained from the solution of equations with initial condition

$$\frac{\partial x(t, q_i)}{\partial t} = u(t, x(t, q_i)),\quad x(t, q_i(t)) = x_i(t),\quad t = t_0,\quad x_i(t_0) = q_i(t_0)$$

Applying the standard procedure of variation of functional (2.8) and eliminating the Lagrangian multiplier, we obtain Euler's equation

$$\frac{\partial u}{\partial q}\left\{\frac{\partial}{\partial q}\left[\left(2\mu\frac{\partial^2 x}{\partial q^2} - \tau\frac{\partial^2 x}{\partial t^2}\right)F\right] - \tau\frac{\partial^2}{\partial t^2}\left(F\frac{\partial x}{\partial q}\right) + 2\mu\frac{\partial^2}{\partial q^2}\left(F\frac{\partial x}{\partial q}\right)\right\} - \frac{\partial x}{\partial q}\left\{\frac{\partial}{\partial q}\left[\left(2\mu\frac{\partial^2 u}{\partial q^2} - \tau\frac{\partial^2 u}{\partial t^2}\right)F\right] - \tau\frac{\partial^2}{\partial t^2}\left(F\frac{\partial u}{\partial q}\right) + 2\mu\frac{\partial^2}{\partial q^2}\left(F\frac{\partial u}{\partial q}\right)\right\} = 0 \tag{2.9}$$

For the known solution $u = x/t$ we have the equation with boundary and initial conditions

$$\frac{\partial}{\partial q}\left[\left(t\frac{\partial x}{\partial t} - x\right)^2\frac{\partial x}{\partial q}\right] = 0,\quad t = t_0,\quad x = q,\quad q = \frac{x_i(t)}{t}t_0,\quad \frac{\partial x}{\partial t} = \frac{x}{t}\quad (i = 1, 2) \tag{2.10}$$

Solving problem (2.10) we obtain

$$x(t, q) = qt/t_0 \tag{2.11}$$

Then $w(x, t) = x/t$, $\Delta(x, t) = t/t_0 \neq 0$.

Solution (2.11) shows that the curvilinear coordinates coincide with the Lagrangian coordinates $x = x_0 t/t_0$.

REFERENCES

1. LISEIKIN V.D. and IANENKO N.N., The method of moving coordinates in gasdynamics. In: Numerical Methods of Continuous Medium Mechanics, Vol.7, No.2, Novosibirsk, VTs SO Akad, Nauk SSSR, 1976.
2. JANENKO N.N. KROSHKO E.A., LISEIKIN V.D., FOMIN V.M., SHEPEEV V.P. and SHITOV Ju.A., Methods for the construction of moving grids for problems of fluid dynamics with big deformations. In: Lect. Notes in Physics, No.59, 1976.
3. JANENKO N.N., DANAEV N.T. and LISEIKIN V.D. On the variational method of constructing grids. In: Numerical Methods in Continuous Medium Mechanics, Vol.8, No.4, Novosibirsk, VTs Akad. Nauk, SSSR, 1977.
4. Physics of Explosion. Ed. Staniukovich K.P. Moscow, NAUKA, 1975.
5. SEDOV L.I., Mechanics of Continuous Medium, Vol.1, Wolters—Noordhoff, 1971.
6. GNESIN V.I., GRODZINSKII V.L., and SOKOLOVSKII G.A., Equations of gasdynamics in a noninertial deformable coordinate system. PMM, Vol.42, No.5, 1978.

Translated by J.J.D.